ELEMENTARY MATH PROJECT

Grade 6

Patterning and Algebra

Key Number Concept 1: Increasing and decreasing patterns, using expressions, tables, and graphs as functional relationships

Sample Week at a Glance

This sample week at a glance kicks off students' formal introduction to increasing and decreasing patterns within Grade 6. Prior to this week, students would have come to the understanding that patterns involve repetition. What repeats can vary. For example, students will have answered open questions such as "Extend the pattern 5, 10, … in as many ways as you can." Possible responses include:

a. 5, 10, 5, 10, 5, 10, …
b. 5, 10, 25, 5, 10, 25, 5, 10, 25, …
c. 5, 10, 15, 20, 25, …
d. 5, 10, 20, 40, 80, …
e. 5, 10, 15, 25, 40, …

(Note that although all of the responses above are mathematically meaningful, students will focus on linear relations--response "c" above--in Grade 6.)

This open question ties together patterns in which elements themselves repeat and patterns in which operations repeat.

Monday	**Before.** Provide students with pattern blocks or access to virtual manipulatives (e.g., Polypad by Mathigon, The Math Learning Center's Pattern Shapes App). Tell a story in which tables and chairs are needed for a large party. Use a green triangle pattern block to represent one table. One person can be seated on each side of a triangular table. In turn, show that three people can sit at one table, six people can sit at two tables, and nine people can sit at three tables. Use a table of values to organize this information. Ask "How many people can sit at four tables? How do you know?" Have students turn-and-talk. Listen for strategies that involve addition (e.g., "nine and three more make twelve") and multiplication (e.g., "four tables of three at each make twelve"). Select students to share these strategies. Annotate their thinking and add on to the table of values. Repeat for 5, 10, 20, 50, and 100 tables. **During.** Tell students that tables come in different sizes: squares, trapezoids, and hexagons, which can be represented by orange, red, and yellow pattern

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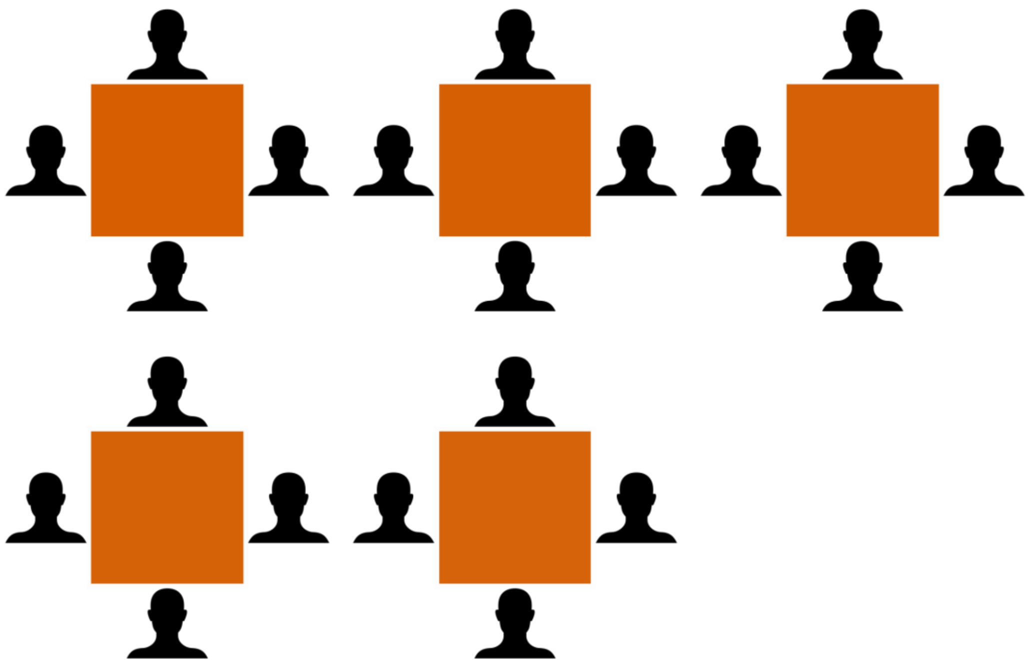
blocks, respectively. (Note that a trapezoid can seat five people.) For each shape, have students build--then draw--the first three figures in the pattern. Ask “How many people can sit at 5, 10, 20, 50, and 100 of each table? How do you know?” Have students solve this problem in pairs. Encourage students to represent their ideas concretely (where practicable), pictorially, and symbolically.


Twenty people seated at five square tables

After. Make connections between repeated addition and multiplication. Highlight that to figure out the number of chairs needed for large numbers of tables, it is more efficient to multiply once than to add repeatedly. This strategy directly relates the two quantities, tables and chairs (or seated people). Return to the case of the triangular tables. Introduce the use of an expression, $3n$, to describe the number of chairs needed for any number of triangular tables. Encourage students to write expressions to describe the number of chairs needed for the remaining shapes.
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Before. Display the following patterns:

6, 11, 16, 21, 26, …
5, 10, 15, 20, 25, …

Ask “What is the same? What’s different?” Possible responses include:

both patterns are increasing
both patterns involve adding five each time
the first pattern starts at six; the second pattern starts at five
the second pattern contains multiples of five; the second does not
the ones digit in both patterns repeats: 6, 1, 6, 1, 6, 1, … in the first and 5, 0, 5, 0, 5, 0, … in the second or even-odd in the first and odd-even in

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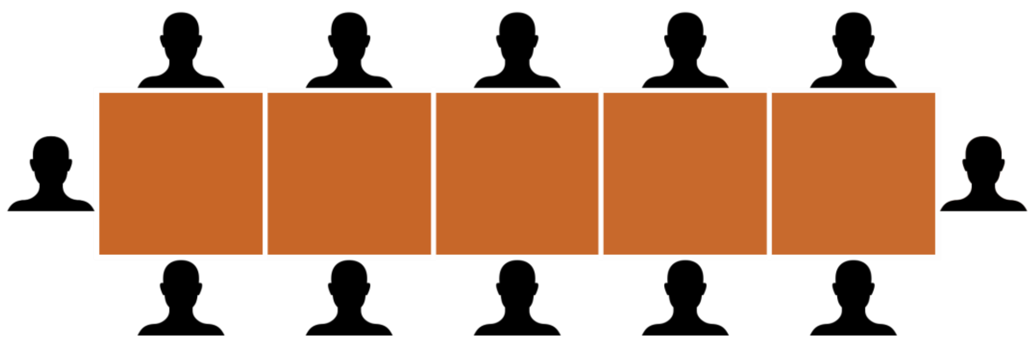
- the tens digit in both patterns is both repeating and increasing: 0, 1, 1, 2, 2, 3, 3, …

Note that this warm-up foreshadows patterns that involve composite rules (i.e., $5n + 1$). See the Instructional Routines page to learn more about Same But Different.

Return to yesterday’s tables and chairs problem. This time, tables can be joined together so that they share one side. As above, model the case of triangular tables, building to 102 people seated at 100 tables.

During. Provide students with pattern blocks or access to virtual manipulatives (e.g., Polypad by Mathigon, The Math Learning Center’s Pattern Shapes App). Invite students, in groups of three, to determine how many people can sit at 10, 20, 50, and 100 of one or all of the remaining shapes. Remind students to represent their ideas concretely (where practicable), pictorially, and symbolically. Ask “How does your pattern grow? Why does it grow in this way?” Monitor. Look for students making use of functional relationships. Encourage students to write an expression to describe each of their increasing patterns. Challenge them to explain the meaning, in context, of the coefficient and constant terms.



Twelve people seated at five square tables

After. Select students to share their strategies for determining the number of chairs needed for 100 square (202), trapezoidal (302), and hexagonal (402) tables. (You might decide to gradually build to 100 tables.) For example: “I chose hexagonal tables. Adding one table adds just four more seats since two sides are ‘lost’ when new and existing tables are joined. There are always two seats at the ends. So, $100 \times 4 + 2 = 402$.” Discuss the expressions for triangular ($1n + 2$), square ($2n + 2$), trapezoidal ($3n + 2$), and hexagonal ($4n + 2$) tables. Highlight that we can use these expressions to efficiently figure out how many chairs are needed for any number of tables.

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<td>Wednesday</td>
<td>Before. Display the following visual pattern:</td>
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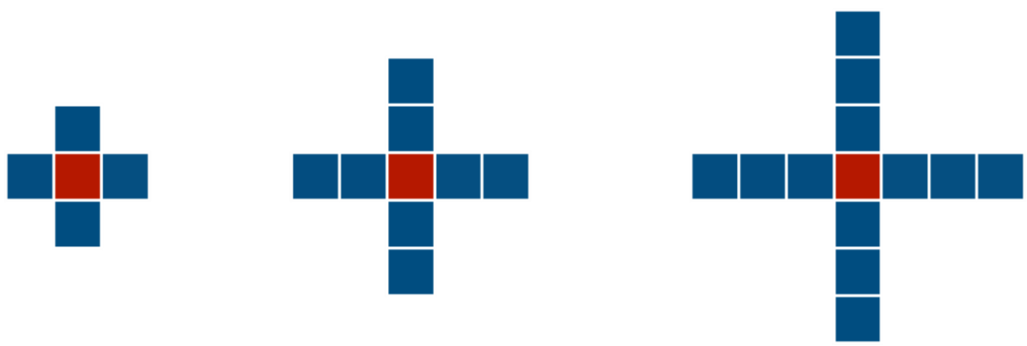
Provide students with colour tiles or access to virtual manipulatives (e.g., Polypad by Mathigon, The Math Learning Center's Whiteboard App). Have students build the first three figures in this pattern.

Ask:

- How do you *see* the pattern growing?
- What comes next? Build it.
- What does Figure 10 *look* like? How many tiles? ($3 \times 10 = 30$)
- What does Figure 100 *look* like? How many tiles? ($3 \times 100 = 300$)
- What does Figure *n* *look* like? How many tiles? ($3n$)
- How many tiles in Figure 43? ($3 \times 43 = 129$)

To learn more about Visual Patterns, see the Instructional Routines page.

During. Display the following visual pattern:

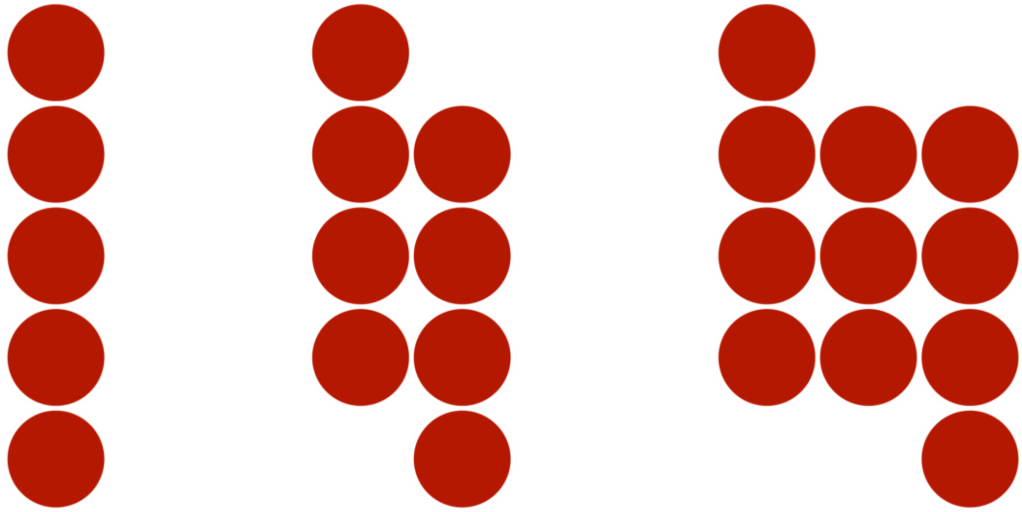


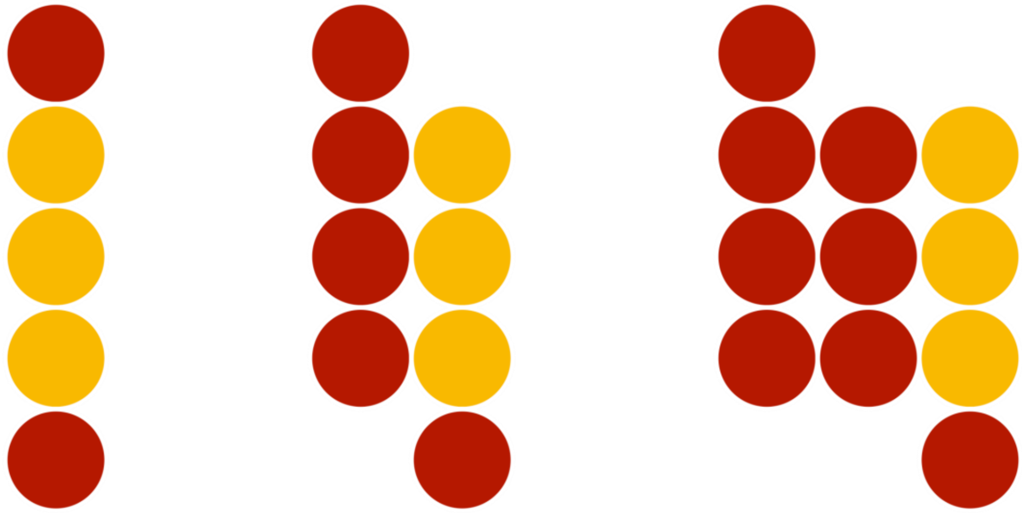
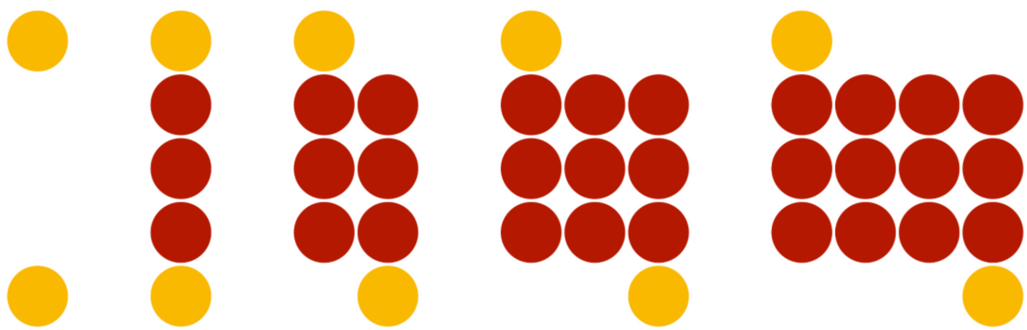
Note that this pattern involves a composite rule.

Invite students, in groups of three, to answer the set of questions above. Encourage students to represent their ideas concretely (where practicable), pictorially, and symbolically. Colour provides some support. If students are "stuck," ask "What stays the same? (one red tile) What changes? (the number of blue tiles)" Again, look for students making use of functional relationships.

After. Select students to share their strategies. For example:

- I noticed that each figure has one red tile. I saw four 'legs' of blue tiles in each figure. The number of blue tiles in each leg matches the figure number. So, the expression is $4n + 1$.
- I see four more tiles each time, one in each direction. Plus, there's always one red tile. So, there are $43 \times 4 + 1 = 173$ tiles in Figure 43.

	Ask “What comes before? What does Figure 0 look like?” (one red tile). Discuss.
Thursday	**Before.** Display the following visual pattern:  Note that this pattern, again, involves a composite rule. Further, students might visualize how it grows in more complex ways--they might describe existing tiles shifting to make room for new tiles. Provide students with counters or access to virtual manipulatives (e.g., Polypad by Mathigon, The Math Learning Center’s Whiteboard App). Have students build the first three figures in this pattern. **During.** Have students, in groups of three, answer the following questions: ● How do you *see* the pattern growing? ● What comes next? ● What comes *before*? ● What does Figure 10 *look* like? Draw it. How many counters? ($3 \times 10 + 2 = 32$) ● What does Figure 100 *look* like? Draw it. How many counters? ($3 \times 100 + 2 = 302$) ● What does Figure n *look* like? How many counters? ($3n + 2$) ● How many counters in Figure 43? ($3 \times 43 + 2 = 132$) Encourage students to represent their ideas concretely (where practicable), pictorially, and symbolically. Challenge students to explain the meaning, in context, of the coefficient and constant terms: “How do you *see* 3? How do you *see* 2?” **After.** Have students share their strategies. Draw out that the 3 and 2 in $3n + 2$ can be seen as what changes and what stays the same, respectively. The expression also reveals the number of tiles in Figure 0.

	 *What changes.*  *What stays the same.*
Friday	**Before.** Read *Two of Everything* by Lily Toy Hong. Introduce the concept of a function and the terms input and output. Like the Huktak's magic pot, a function determines a particular output for a particular input. Revisit the part of the story when five coins are put into the pot and doubled. Create a table of values to record this information. Together, continue to add rows to the table of values using this doubling rule: "What if the Haktaks put 10 coins into the pot? 20? 50? 100? 43?" Introduce a new function (or "magic pot"), such as $5n$ or $2n + 3$. Once again, build a table of values together. **During.** Provide students with a table, like the one below. Have students, in groups of three, determine the rule for each "mystery function." When students are ready to guess your rule, invite them to test their prediction by providing you with a new input. After providing students with the output--and if they are confident--have them guess your rule. Ask "How did you determine the rule?" Repeat with a different table of values. Challenge students to create their own "mystery function" for their partners if needed.

input	output
1	7
2	14
3	21
4	28
5	35
20	140
43	301

input	output
0	1
1	5
2	9
3	13
4	17
10	41
50	201

input	output
1	5
2	8
3	11
4	14
5	17
10	32
36	110

$7n$; $4n + 1$; $3n + 2$

After. Select students to share their strategies for determining the rule for each function. Students may use words (e.g., "the output is two more than triple the input") or expressions (e.g., "$3n + 2$"). Highlight that the coefficient is related to the increase within the table (or rate of change) and that the constant is related to the output for an input of zero.

Next, students will explore increasing patterns in contextualized situations that, unlike the tables and chairs explorations above, cannot be modelled using manipulatives. For example, the relationship between cost and number of items or the relationship between savings and time. As above, students will describe patterns, predict what comes next and what comes 'way down the line,' and generalize functional relationships. Contextualized situations provide a way to introduce decreasing patterns. For example, students might explore the following: "Keira receives a $50 gift card to a local bubble tea shop. Each day, they order their favourite drink, which costs $6." Students might then be asked to determine the balance of the card after 1, 2, 3, 8, or *n* visits.

Students will then be introduced to representing increasing and decreasing patterns using graphs. highlights relationships visually. Visual patterns and contextualized situations, such as those above, will be revisited. Students will make connections between graphs and other familiar representations (i.e., words, tables, expressions). Students will learn that graphs visually convey the mathematical ideas (e.g., what changes, what remains the same) within these other representations.