ELEMENTARY MATH PROJECT

Grade 7
Patterning and Algebra
Key Number Concept 1: Discrete Linear Relations
Sample Week at a Glance

Prior to this week, students would have explored number patterns, and specifically linear patterns (going up or down by a constant amount) by investigating examples and non-examples. They also would have reviewed the key concepts, representations (including visual patterns), and vocabulary involving linear relations from Grade 6.

<table>
<tr>
<td>Monday</td>
<td>
Focus: Extending increasing linear relations using a visual pattern and table of values
Before:

Do a Same But Different routine to explore an increasing vs a decreasing linear pattern. For example:

3, 7, 11, 15, …
48, 44, 40, 36, …
How are they the same? [e.g., they both change by 4]
How are they different? [e.g., one is increasing and the other is decreasing]
Is there a number that will be in both patterns? [e.g., no, because one only has odd numbers and the other only has even numbers]

During:

Present an increasing linear visual pattern such as:

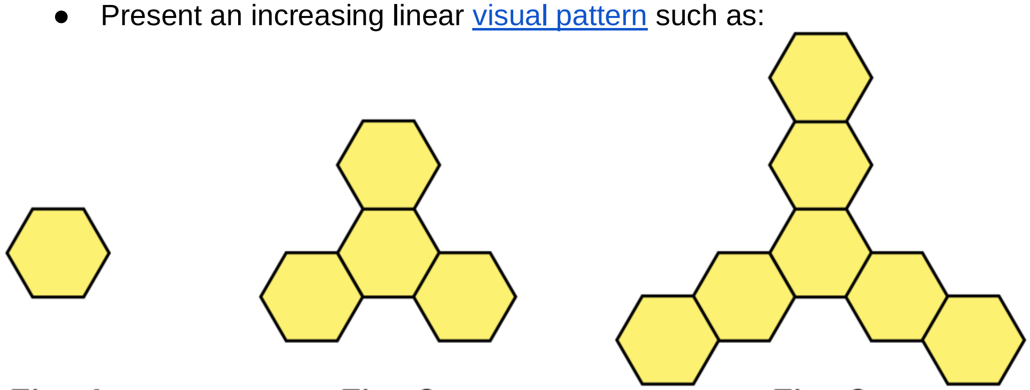


Fig. 1 Fig. 2 Fig. 3

Students may work in pairs or small groups.
Using pattern blocks (or the polygon tools from Polypad), ask students to build the next two figures. Discuss how the pattern is growing.
As a class, create a table of values of the total number of hexagons in each of Figures 1 to 5. Ask the class what they notice.

</td>
</tr>
</table>

Figure	Hexagons
1	1
2	4
3	7
4	10
5	13

- Ask the class if they can figure out how many hexagons would be in Figure 20.
- Circulate among the groups and observe the different strategies. Prompt for thinking with questions such as, “How do you know?”, “Is there another strategy you could try?”, etc.

After:

- Ask how many hexagons are in Figure 20. [58]
- Have some groups share their strategy for determining how many hexagons are in Figure 20, ideally in a sequence that progresses. For example:
    - We built each figure by adding on 3 hexagons each time. When we got to Figure 20 we needed 58 hexagons.
    - We extended the table by adding 3 each time to get 58.
    - To go from Figure 5 to Figure 20 we would need to add 3 hexagons 15 more times, which is 45 more hexagons. 13 + 45 = 58.
    - There’s the one hexagon in the center, and then 3 branches of hexagons which are one less than the figure number. So 3 x 19 + 1 = 57 + 1 = 58
    - If we do 3 times the figure number for each branch, we need to subtract 2 hexagons because we count the center hexagon 2 extra times. 3 x 20 – 2 = 58
- As each strategy is shared, ask the class how this strategy connects to the earlier strategies. Note especially the transition from extending the pattern one figure at a time to using more efficient strategies that *jump* to Fig. 20.
- Ask the class to use one of the strategies to determine the number of hexagons in a figure way down the line. They can choose which figure.
- Have the class work on one or more other such visual patterns (increasing linear patterns), or some groups may wish to create their own.
- Explain that next class they will work on figuring out an algebraic expression for such patterns.

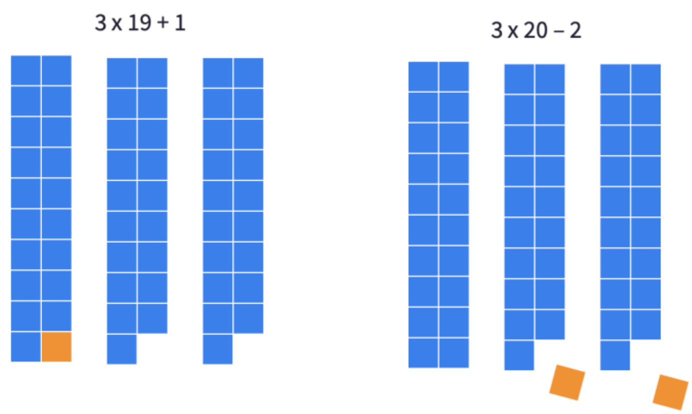
Tuesday

Focus: Representing linear relations with an expression using the visual pattern and table.

Before:

- Show the first 3 figures from the visual pattern from yesterday.
- Write down the following 2 expressions to describe two of the strategies for Figure 20 from yesterday:
    - 3 x 19 + 1 = 58

        - 3 x 20 – 2 = 58
- Have a class discussion with questions such as:
    - How does each equation represent how many hexagons are in Figure 20?
    - How can you show that both expressions are equivalent?
        - Either visually or numerically, one can see that with both expressions, you end up with one group of 20 and two groups of 19.



        - In the visual pattern, this makes sense because we can see visualize one branch of the figure number, and two branches that are one less than the figure number.
    - Point out to the class that they will be focusing today on expressions like the second one because it relates to the figure number directly.

During:

- Have the class work in pairs or small groups.
- Provide a different visual pattern (an example is below) for them to explore. Alternatively you may wish to provide more than one and ask them to choose. The patterns could be of different complexities.

Fig. 1 Fig. 2 Fig. 3

- Ask them to make a table of values.
- Ask them to explore how many squares would be in Figure 50, encouraging them to think both visually (how to visualize what Fig. 50 looks like) and numerically (how they could jump down the table of values).
- Circulate among the groups and observe the different strategies. Prompt for thinking with questions such as, “How do you know?”, “Where do you see the figure number in each figure?”, etc.

After:

- Ask how many squares are in Figure 50. [197]
- Ask how does the pattern grow. [4 squares are added each time]

- Write out a table of values, but insert a column for multiplying the figure number by 4 to represent the repeated addition of 4. Leave room for a row before Figure 1.

Figure (n)	4 x n	Number of Squares
1	4	1
2	8	5
3	12	9
4	16	13
5	20	17
50	200	197

- Ask how the third column compares to the second column. [the third column numbers are 3 less than the second column numbers]
- Add a row and write the figure number as *n.* Ask how they could **generalize** this pattern using an **expression**.

Figure (n)	4 x n	Number of Squares
1	4	1
2	8	5
3	12	9
4	16	13
5	20	17
50	200	197
n	$4n$	$4n – 3$

- Ask them to test the expression with other figure numbers. For example, 4 x 5 = 20 – 3 = 17
- Ask them what if we extended the pattern backwards. How many squares would be in Figure 0? Record in the table.

Figure (n)	4 x n	Number of Squares
0	0	– 3
1	4	1
2	8	5
3	12	9
4	16	13
5	20	17
50	200	197
n	$4n$	$4n - 3$

– 4
– 4
– 4

- Ask them what they notice about the number for Figure 0. [It's the same as the number they put after the 4*n*.]
- Ask them if there is a way to see this –3 in the visual pattern? [If we add in 3 'negative' squares to each figure, we can see that the figure number says how many groups of 4 squares there are.

Fig. 1 Fig. 2 Fig. 3

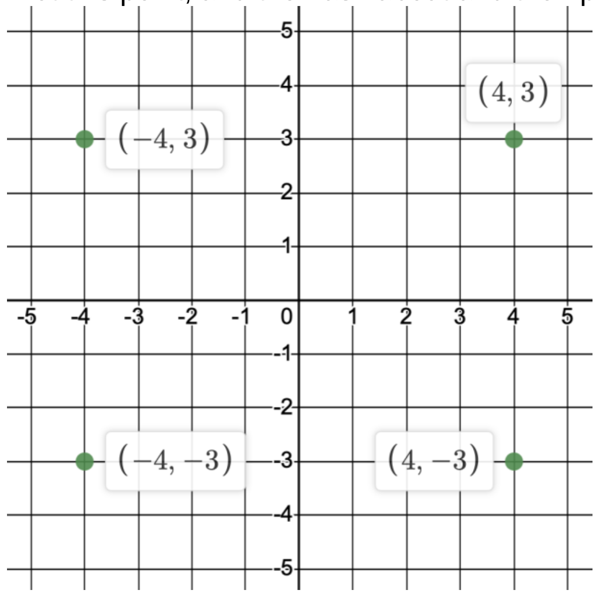
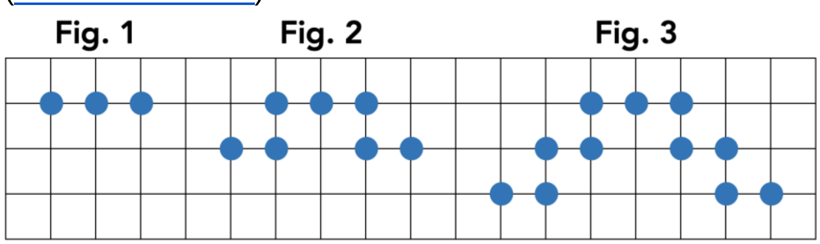
- Explain that the –3 is called the **constant** of the expression, because its value does not change but is the same for each figure. The 4 in 4*n* is called the **rate of change** because it describes how the pattern is growing.
- Have the class work on one or more other such visual patterns (increasing linear patterns), or to try numerical patterns such as 2, 7, 12, … For each one, ask them to determine an expression that generalizes the pattern, and to identify the rate of change and the constant.

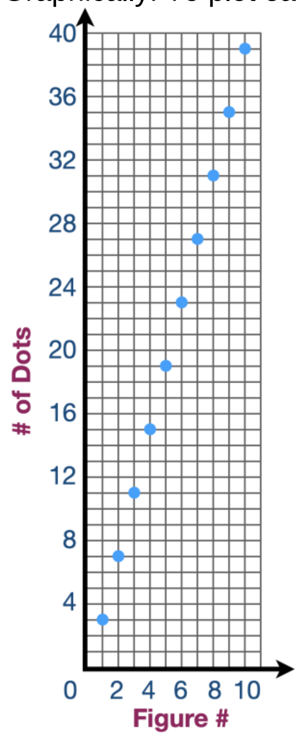
Wednesday

Focus: Graphing linear relations, and making connections between representations

Before:

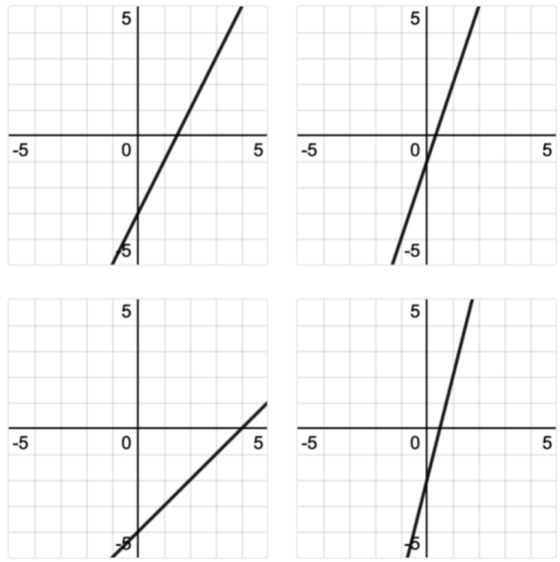
- Display a coordinate grid but just show the first quadrant.
- Write down positive coordinates such as (4, 3). Ask the class where they would plot the point on the grid. You may also use a deck of cards to get random numbers, but just use the black cards at first.
- Repeat the process for another point or two.
- Next write down (–4, 3) and ask them how they think we would be able to plot this point? [The x-value is now negative so we can extend the number line to the left]
- Extend the grid to the left and plot the point.
- Next write down (3, –4) and ask them how we could plot this point. [We

	can extend the y-axis numberline down]. ● Plot this point, and then ask about and then plot the point (–4, –3).  ● Record the points into a table of values. ● Provide students with grid paper and a deck of cards. Have them work in pairs to practice plotting points in 4 quadrants by randomly drawing two cards for each point. The red cards can represent negative integers. Ask them to record their points in a table of values as well. During: ● Provide a visual pattern (either a new one or you could use one from earlier in the week). This one below is adapted from Fawn Nguyen (Visual Pattern #75)  ● Have students work in groups. ● Ask them to create a table of values, and then to graph the table of values. ● Ask them to figure out how many dots would be in Figure 10, and how they could make sense of this number: ○ Visually using the visual pattern ○ Numerically using the table of values ○ Graphically by extending their graph ○ Algebraically by using an expression After: ● Have the class share how each representation can be used as a strategy for figuring out how many dots would be in Figure 10. ○ Visual Pattern: ■ There are 9 rows of 4 dots, plus the row of 3 dots on the top, for a total of 39 dots.

	■ There are 10 groups of 4, but the top group is missing one, so 10 x 4 – 1 = 39 ○ Table of Values: Each row goes up by 4 each time. Figure 3 has 11 dots, and in 7 more rows will add 28 more dots for a total of 39. ○ Graphically: To plot each next point we go one over and 4 up.  ○ Expression: We reasoned that the expression was 4*n* – 1. So for figure 10, 4 x 10 – 1 = 39 ● Discuss what was similar or different between the strategies, and which one they prefer for determining the answer. Using the expression is probably the simplest, but to figure out the expression you need to use another one of the representations. ● As an exit ticket, ask the students to describe where they see the rate of change (4) and constant (–1) in each representation.
Thursday	Focus: Extending linear relations in both directions Before: ● Write down 2 increasing patterns. For example: ○ 1, 3, 5,... ○ 2, 5, 8, … ● Say to the class, “Both patterns have the number 5. What other numbers are in both patterns?” ● Give students time to explore in pairs. ● Share what they found out. [The numbers they have in common are 5, 11, 17, 23, …] ● Ask why they think a common number happens every time they go up by 6? [The first pattern increases by 2 and the second pattern increases by 3, and the LCM of 2 and 3 is 6.]

	● What if we extended each pattern to the left? [5 was common, so we can go backwards by 6 each time to get –1, –7, –12, …]
	During:
	● Have the students work in pairs or small groups.
	● Present this problem to the class: A pattern has the numbers 2 and 8 in it. What could the pattern be?
	● Explain to the class that this is an open question, so there are many possible correct answers. They may use any representation they find helpful.
	● Ask them to determine an expression for each pattern they find.
	After:
	● Have the class share their patterns and their strategies. Some may use a graph or a table. Below is a strategy using patterns written as sequences:
	    ○ We realized we could go from 2 to 8 by increasing by 1, 2, or 3 or 6. So our patterns were:
	        ■ **2**, 3, 4, 5, 6, 7, **8**, 9, … $n + 1$
	        ■ **2,** 4, 6, **8**, 10, … $2n$
	        ■ **2,** 5, **8**, 11, … $3n - 1$
	        ■ **2, 8,** 14, … $6n - 4$
	● Consider the pattern that increases by 3. It could have different starting numbers:
	    ○ 2, 5, 8, 11, …
	    ○ –1, 2, 5, 8, …
	    ○ –4, –1, 2, 5, 8, …
	    ○ –7, –4, –1, 2, 5, 8, …
	    ○ Ask students to determine an expression for each pattern. What do they notice?

Friday	Focus: Practice exploring and representing increasing linear relations, including connecting representations Before: ● Show students 4 graphs and 4 expressions. For example:



$n - 4$

$3n - 1$

$4n - 2$

$2n - 3$

- Have a class discussion about which expression goes with each graph, and how do they know. Some may suggest using the rate of change and others may suggest using the constant.

During:

- Create a set of cards of four or five linear relations with several representations of each, which could include:
  - Expression
  - Table of Values
  - Graph
  - Visual Pattern
- Have students work in groups. Give each group a set of the cards.
- Ask the students to sort the cards into different linear relations by matching representations.
- When done, a group can share and compare their sets with another group.

After:

- Have a discussion about what strategies they used to match the representations. Were there representations that they chose to match up first? For example:
  - We started by matching tables of values with graphs because we could easily see the coordinates in the table.
  - We looked at the rate of change first and matched representations that had the same rate of change.
- Groups could then create their own card set and share it with another group.

The following week students will explore linear relations with decreasing patterns. They should be able to move the different aspects of this concept at a quicker pace given what they have learned about increasing patterns. Students would also explore contextual problems involving both increasing and decreasing patterns.