ELEMENTARY MATH PROJECT

Grade 7
Patterning and Algebra
Key Number Concept 2: Two-Step Equations
Sample Week at a Glance

This sample week describes what could be done during the first week of this unit.

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<td>Monday</td>
<td>Focus: Using contextual problems to make sense of two-step equations
Before:

Have students explore single-operation contextual problems taking place in a dollar store (assuming whole number prices only). For example (choose “items” that are interesting for your students):

2 different items cost a total of $14. If one item costs $6, how much does the other item cost?
4 items cost $8. How much does one item cost?
After a $6 item was put back on the shelf, the total was now $18. What would the total have been before the $6 item was returned?
What was the total cost of 3 items that cost $4 each?

Have students share their ideas about which operation made sense for each scenario. Because of how the operations are related, the same scenario may be looked at as involving addition or subtraction. Similarly, the same scenario may be looked at as involving multiplication or division.

During:

Have students work in pairs or small groups.
Tell students they may use whatever approach that makes sense to them to solve the problem, include concrete materials, pictures, or symbolically.
Present a problem such as: “At a dollar store, a sports drink costs $3. Jose bought one sports drink and 4 nutrition bars. His total before taxes was $11. How much does one nutrition bar cost?”
For groups that finish before others, you could give them further things to do such as:

Is there a different strategy or representation you could use?
Write a similar problem using items that you choose, and give it to another group to solve.

After:

Have some groups share how they solved the problem. Ideally select them in order of complexity. For example:

We used 11 counters. 3 of them were the sports drink so that left 8 counters. We put those 8 counters into 4 groups to represent the nutrition bars, so each bar cost $2.

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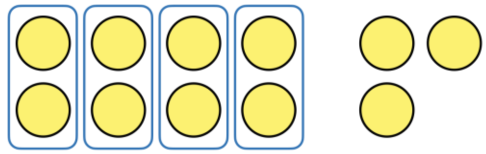

- ○ We subtracted $3 from $11 to get $8. We then divided the $8 by 4 to get $2 each.

- Discuss how the different approaches all used similar thinking, and we call this **algebraic thinking**. In this case it made sense first to subtract 3, then to divide by 4.
- Even though they will formalize an algebraic approach before the end of the week, it is helpful to represent their approaches symbolically even at this stage.

$$4n + 3 = 11$$ Subtract $3 sports drink

$$4n = 8$$ Divide to determine cost of one bar

$$n = 2$$

- As a closing task, you could set up something like this problem but with larger numbers. Put 11 counters into each of 3 bags. Put 12 counters scattered outside the bags. Tell the class that you have 45 counters. You put the same number of counters into each bag, and there are 12 counters left over. How many did you put into each bag?

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Tuesday

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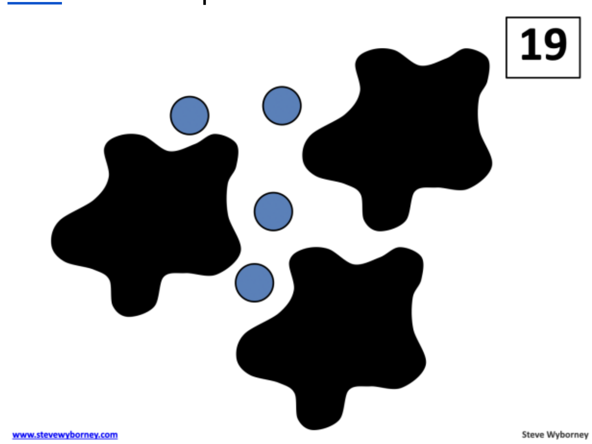
Focus: Solving equations concretely/pictorially using algebra tiles
Before:

- Splat! Is a powerful routine connecting number sense and algebraic thinking. Do one or two Splat! activities. Choose from the Multiple Splat sets. For example:



- Have students explain their thinking for how they know how many dots are under each splat.

During:

- This learning experience uses algebra tiles. Though these may be new to many of your students, they are no different from any of their other concrete explorations in which they used some object to represent an unknown value. Students will use the x-tiles to represent the variable, and the units tiles to represent the numbers.
- This activity uses algebra tiles and a balance from Polypad.

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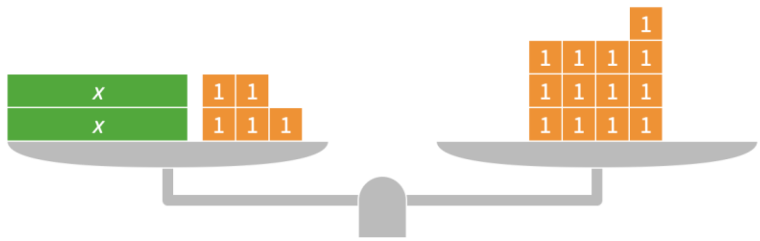
Alternatively, you could use concrete algebra tiles. You could spread this over two days if you wanted one day just to explore using algebra tiles before using the balance on the second day.

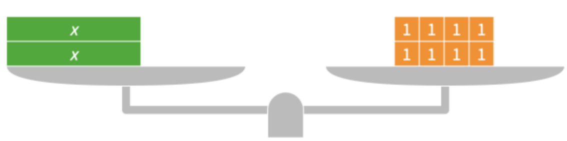
- Have students work individually, or in pairs. After each prompt below, monitor students progress to make sure they are modelling the situation correctly. You may wish to do a quick demo before hand so that they see how the balance scale tool works.
    - Under the Algebra menu, bring in a balance scale.
    - On the left side of the balance, model 2*x* + 5 using algebra tiles
    - Build 13 on the right side using unit tiles.
    - Balance the scale by clicking on the fulcrum (centre part) and clicking on Balance.



    - Tell students to remove one of the unit tiles from one side. What do they notice? How could they restore the balance? [They could put the unit tile back, but the idea intended here is to remove a unit tile from the other side.]
    - Tell students to continue manipulating the tiles in order to figure out what the value of the *x*-tile is while keeping the scale balanced.
    - Once the solution makes sense to them, they could try some other equations.

After:

- Show the equation modelled using Polypad on a balance scale. Ask students to guide you through the process of solving the equation. As you do each step, represent the process algebraically.
    - Remove five tiles from each side.
    - Put the right side into two groups
    - Each *x*-tile balances with 4 unit tiles, so *x* = 4



$$2x + 5 = 13$$
$$-5 \quad -5$$
$$2x = 8$$
$$\div 2 \quad \div 2$$
$$x = 4$$

- Explain to the class that this process is called **isolating the variable**.
- Give students a choice of some other equations to model and solve using the algebra tiles and balance scale.
- As an exit ticket or journal prompt, ask students to compare the process they used today with how they solved an equation like this before today. What is similar? What is different?

Wednesday

Focus: Formalizing preservation of equality using inverse operations as a strategy to solve two-step equations

Before:

- Present the equation:

  $$\boxed{} + 12 = 22$$

- Have the class share what number goes in the box and how they know.
- Write 2*n* inside the square.

  $$\boxed{2n} + 12 = 22$$

- Do a think-pair-share on how changing the unknown to 2*n* affects the solving process.
  - We had to do an extra step. We know the box equals 10, but there are two *n*'s so each *n* must equal 5.
- Explain to the class that what they have experienced with solving these types of equations is that it makes sense to isolate the term with the variable first.

During:

- Have students work in pairs or small groups.
- Provide a few equations for them to solve (of the form *ax* + *b* = *c*). Use different letters for each equation to indicate that a variable can use any letter.
  - For example: 5*c* + 14 = 29 (the numbers chosen make a concrete approach less practical).
- Encourage them to see if they can solve them without using concrete or pictorial models.
- Observe the groups as they work on the equation. If needed, suggest drawing a box around the variable term (as was done in the *Before*) and figuring out the value of the box first.

After:

- Have the class share their strategies for solving each equation. For the example above:
  - 29 – 14 = 15, so the 5*c* must equal 15. So we divided 15 by 5 to get *c* = 3
- Record the steps to model the process symbolically for one of the equations:

  $$5c + 14 = 29$$
  $$-14 \quad -14$$
  $$5c = 15$$
  $$\div 5 \quad \div 5$$
  $$c = 3$$

- Consolidate the learning of this process by identifying the key concepts. The thinking through these steps is the same as they experienced when using the balance scale model.
  - To solve the equation, we needed to isolate the variable, i.e., get the variable on its own.

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To keep the equation balanced, whatever operation we did to one side, we needed to do to the other.
Notice that the operation on both sides each time was the inverse operation.

When we had + 14 we had to subtract 14.
When we had 5n, we needed to divide by 5.

Explain to students that using inverse operations is a helpful strategy, but not the only strategy. Most strategies still involve the same kind of algebraic thinking, even if they are carried about using different approaches.
In addition to having students practice solving more equations, you could have them build their own equations and have a classmate solve them.

Pick a letter to represent the variable.
Give the variable a value, e.g., $w = 6$
Choose a coefficient, e.g., $3w$
Choose a constant, e.g., $3w + 7$
Evaluate their expression for the value they chose for the variable, e.g., $3 \times 6 + 7 = 25$
The equation is $3w + 7 = 25$

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<td>Thursday</td>
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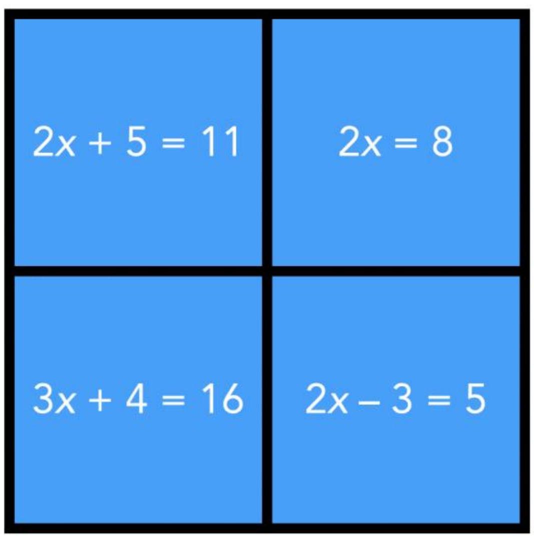
Focus: Solving other forms of equations (you may wish to omit this if you’re satisfied focusing on just solving $ax + b = c$ forms of equations).
Before:

Do a Which One Doesn’t Belong routine



Have students share their ideas. Students may describe several different reasons for each. Here are some sample answers:

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• x = 3, but the others have x = 2	• Only has one operation; the others have two
• Has 3x, but the others have 2x	• Only one with subtraction

- If not all of the equations were solved, have students solve each equation using whichever strategy or representation makes sense for them. They may struggle to solve the bottom right equation algebraically. That will be part of the focus of today's lesson.

During:

- Have the class work in small groups.
- Present 3 equations to the class that use similar numbers but have different structures. For example:

$$2x + 5 = 13 \qquad 2x - 5 = 13 \qquad \frac{x}{2} + 5 = 13$$

- Ask them to solve the equations, and to reflect on how the solution was process compared between the equations.
- As with the previous day's lesson, if needed you may suggest that they put a box around the variable term and figure out the value of the box.

After:

- Have the class guide you through solving the equations. Rather than do one equation at a time, go through each step for all equations so that the similarities and differences can be noted.

$2x + 5 = 13$	$2x - 5 = 13$	$\frac{x}{2} + 5 = 13$
$-5 \quad -5$	$+5 \quad +5$	$-5 \quad -5$
$2x = 8$	$2x = 18$	$\frac{x}{2} = 8$
$\div 2 \quad \div 2$	$\div 2 \quad \div 2$	$\times 2 \quad \times 2$
$x = 4$	$x = 9$	$x = 16$

- The second equation was different because of subtraction. So we needed to use addition as the inverse operation.
- The third equation was different because of division. So we needed to divide as the inverse operation.
- Provide students some additional equations of each type to solve for

	practice. They may choose alternative strategies if they so choose.
Friday	Focus: Connecting solving equations to linear relations. This lesson would be optional, but it is always meaningful to make mathematical connections between concepts. Before: ● To activate their prior knowledge about linear relations, have students match expressions with linear patterns. For example, match these: 1, 5, 9, 13, … $3n - 1$ –1, 2, 6, 10, … $4n - 3$ 2, 5, 8, 11, … $3n - 4$ 3, 7, 11, 15, … $4n - 1$ ● Some ideas that may emerge include: ○ The rate of change tells us the coefficient of the variable. For example, 3*n* indicates which patterns increase by 3 each time. ○ We can figure out the constant by determining which number comes before in each pattern, because then the variable would have a value of 0. During: ● Select one of the matched patterns from the *Before.* For example: 2, 5, 8, 11, …, 3*n* – 1 ● Remind students of the terminology. In this pattern the first term is 2, the second term is 5, and so on. ● Have students work in pairs or small groups. ● Ask them to figure out which term in this pattern is 62. ● If any groups are finished before others, encourage them to see if they can use more than one strategy. They may also try a problem like, “Do any of the patterns from the *Before* have the number 50 in it? If so, which pattern(s), and which term?” After: ● Have some groups share their answer and strategy. Begin with strategies that do not involve using an equation first. For example: ○ We kept adding 3 until we reached 62. It ended up being the 21st term. ○ The 4th term is 11, so we had to add 51 more. That’s adding three 17 more times, so it would be the 21st term. ○ We used an equation, 3*n* – 1 = 62, and solved for *n.* We got *n* = 21. ● Apply the using of an equation strategy to solve a related problem. For example, which term in –1, 3, 7, 11, …, 4*n* – 3 is 61? ● Ask the students to note how the variable means something a bit

	different between an expression and an equation. ◦ In an expression, a variable can be any value. ◦ In an equation, a variable represents a specific, but unknown, value. • Have students explore similar problems, but include decreasing patterns and/or visual patterns. You may also wish them to explore contextual problems or finding the coordinates of a distant point on a graph.

The week above is sufficient to fulfill the curricular expectations for solving two-step equations. Students would still benefit from revisiting these concepts throughout the year through short routines (you could do an Equation Talk in the same way as a Number Talk) or warm-ups, including contextual problems.